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REFINEMENT OF WALL-TURBULENCE HYPOTHESES

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The correspondence between the hypotheses and the experimental data is examined. The distribution functions of the Prandtl mixing length near a smooth wall and the Komogorov turbulence scale in pipe flow are refined.

Present-day methods for the calculation of heat and mass transfer associated with the turbulent flow of a liquid or gas in pipes and boundary layers are based on semiempirical theories using a certain linear turbulence scale. In turn, various hypotheses are used to determine the turbulence scale, but they have not been corroborated by direct comparison of the calculated and experimental values of the scale. It is customary to test only the correspondence of the proposed hypotheses with the measured velocity profile, and this approach admits differing, occasionally mutually exclusive assumptions in the face of the scatter of the experimental points.

In the case of the mixing length l introduced by Prandtl, e.g., neither of the more precise relations of Prandtl [1]

$$l_+ = \varkappa y_+ \tag{1}$$

or Rotta [2]

$$l_+ = \varkappa (y_+ - y_{1+}) \tag{2}$$

has been determined, where y_{1+} is a quantity roughly equal to the dimensionless thickness of the viscous sublayer. The discrepancy of the results of calculations according to these relations near a wall (for $y_+ \approx 30$) attains 30-50%, which does not meet the accuracy requirements of engineering computations.

Neither has the transition from the quadratic law predicted by L. D. Landau and V. G. Levich [3] for the variation of the mixing length l near a wall to a linear function been determined. According to Sherstyuk's hypothesis [4], the quadratic function is replaced by the linear law (2) at a dimensionless distance from the wall $y_+ = 15$. According to Van Driest's hypothesis [5], this transition is a smooth exponential, which approaches the linear law (1) asymptotically as the dimensionless distance y_+ increases:

$$l_+ = \varkappa y_+ [1 - \exp(- y_+/A_+)]. \tag{3}$$

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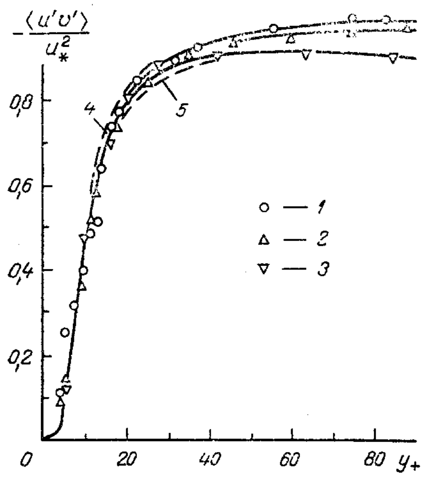

Fig. 1

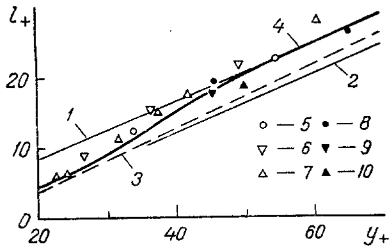

Fig. 2

Fig. 1. Distribution of relative turbulent tangential stress near a smooth wall. 1-3) Experimental data of [6]: 1) in a boundary layer; 2) in a pipe at Re = 500,000; 3) in a pipe at Re = 50,000; 4, 5) calculated curves according to the respective hypotheses of Sherstyuk [4] and Van Driest [5].

Fig. 2. Distribution of the dimensionless mixing length near a smooth wall. 1-4) Calculated according to Eqs. (1)-(3) and (6), respectively; 5-7) in a boundary layer for air, water, and transformer oil, respectively; 8-10) pipe flow of water at Re = 23,300, 43,400, and 105,000, respectively.

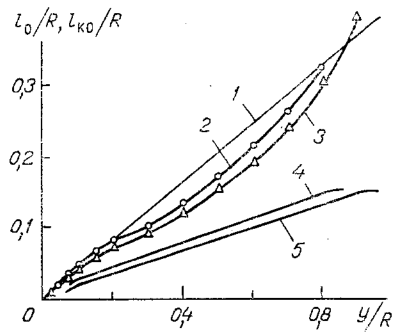

Fig. 3. Distribution of the reduced mixing length and reduced turbulence space scale along the pipe radius. 1) According to Eq. (1) for $\varkappa = 0.41$; 2, 3) l_0/R according to Eq. (4) and the experimental data of [9]; 4, 5) l_{K_0}/R according to Eq. (10) and the experimental data of [9, 13]; 2, 4) Re = 43,400; 3, 5) Re = 396,000.

To compare the latter two hypotheses we recruit the experimental data of Klebanov and Laufer [6] on the distribution of the relative turbulent tangential stress $-\langle u'v'\rangle/u_*^2$ near a smooth wall in a boundary layer and in pipes; these results are represented by the points in Fig. 1.

The deviation of the dashed curve 4 from the experimental points in the interval $10 < y_+ < 25$ indicates that Sherstyuk's hypothesis yields a 10-12% overestimation of the calculated values of the mixing length. The exponential function (3) gives better agreement with the experimental points in this inverval.

On the other hand, Van Driest's hypothesis [5] is not sufficiently accurate in the inverval $20 < y_+ < 40$. The dashed curve 5, which is plotted on the basis of Eq. (3) in application to turbulent flow in a pipe of diameter 247 mm at Re = 50,000 and for the values of the parameters $\varkappa = 0.4$ and $A_+ = 27$ recommended in [5], lies somewhat below the corresponding experimental points.

The function (3) can be improved by comparing the values of the mixing length calculated according to the experimental data with this function. Inasmuch as the direct graphical or numerical differentiation of the experimental velocity distributions near the wall induces inadmissible errors, the computational method proposed in [7] is applicable in the present study.

If we assume that the exponential function (3) goes over smoothly to the linear function (2) at a distance y_+ equal to the value of the parameter A_+ in Eq. (3), the dimensionless distance in Eq. (2) acquires the value $y_{1+} = A_+/e$, and the parameter A_+ enters into the equation of a logarithmic velocity distribution [7]:

$$u_+ = \frac{1}{\varkappa}\ln\left(y_+ - \frac{A_+}{e}\right) + \frac{1}{2\varkappa^2 y_+} + 0.205\,A_+ + 0.32. \tag{4}$$

When the stated assumption is valid, the parameter A_+ in Eq. (4) must remain constant in the interval where Eq. (2) is valid, i.e., at distance y_+ greater than A_+.

It has been shown [7] that the quantity A_+ in (4), calculated according to the experimental data, decreases abruptly near a smooth wall (for $y_+ > 25$) approximately from 30 to 24-26 as the dimensionless distance increases to $y/R = 0.1$. Consequently, the law governing the variation of the mixing length l_+ does not coincide with the function (2) in this interval.

In calculating the mixing length in the present study, we use an equation deduced from the representation of the turbulent tangential stress according to Prandtl's hypothesis [1]:

$$\frac{du_+}{dy_+} = 0.5\left(\sqrt{1 + 4l_+^2\,\tau/\tau_w} - 1\right)/l_+^2. \tag{5}$$

The velocity derivative du_+/dy_+ is determined by differentiating Eq. (4):

$$\frac{du_+}{dy_+} = \frac{\partial u_+}{\partial y_+} + \frac{\partial u_+}{\partial A_+}\,\frac{dA_+}{dy_+}.$$

The derivative dA_+/dy_+ is determined by differentiating the computer-generated analytical expression for the distributions A_+ [7].

To calculate the mixing length we recruit experimental data on the velocity distribution in a boundary layer associated with the flow of air, water, and transformer oil [8] and also in the stabilized pipe flow of water [9]. The results of the calculations, which are shown in Fig. 2, show that mixing length l_+ grows more rapidly than according to the exponential function (3) to values consistent with Eq. (1). The condition of good agreement with the experimental data of [6] shown in Fig. 1 is established by the empirical equation

$$l_+ = \varkappa y_+\{1 - \exp\left[-y_+/(A_+ - 0.25y_+^2/A_+)\right]\}, \tag{6}$$

where the parameters have the values $\varkappa = 0.41$ and $A_+ = 30$, which have been determined [10] on the basis of the principle of maximum stability of turbulent flow.

The solid curves in Figs. 1 and 2 represent the results of calculations based on Eq. (6). Figure 2 shows that the function (6) correctly mirrors the behavior of the mixing length in the interval $20 < y_+ < 60$. For $y_+ < 10$ it practically coincides with the exponential function (3), and for $y_+ = 60$ it makes a smooth transition to the straight line (1). It is essential to note that this same quantity $y_+ = 60$ delimits the wall zone characterized by the direct dissipation of energy from the average flow as heat [6]. The parameter A_+ in Eq. (6) also has physical significance, in that it is equal to the total dimensionless thickness of the viscous sublayer and the transition (buffer) zone.

The linear function (1) is valid in a boundary layer, strictly speaking, under the condition that the tangential stress τ can be assumed to be approximately equal to the tangential stress τ_w at the wall. In pipe flow, where the tangential stress τ is proportional to the distance from the pipe axis, Eq. (1) is applicable only in a limited wall zone.

A more general condition corresponding to a logarithmic velocity distribution in pipes can be obtained by representing the dependence of the mixing length on the distance in the form of an equation involving the ratio of the tangential stresses:

$$l_+ = \varkappa y_+ \sqrt{\tau/\tau_w}\,. \tag{7}$$

At a sufficiently large distance from the wall (for $y_+ > 30$), such that the 1 can be neglected under the square root sign, the substitution of expression (7) therein enables us to cancel the tangential stresses, whereupon the integration of Eq. (5) yields an equation for a logarithmic velocity distribution.

In order to expand the domain of validity of the function (1) it is useful to introduce the concept of the reduced mixing length l_0, which corresponds to the condition of a constant tangential stress:

$$l_0 = l\sqrt{\tau_w/\tau}\,. \tag{8}$$

The dependence of l_0 on the distance y is linear in the flow regions where the logarithmic velocity distribution is valid.

The latter assertion is confirmed by calculating the values of the reduced mixing length corresponding to the experimental data of Nikuradse [9] on the velocity distribution in pipes. The results of the calculations, which are represented by curves 2 and 3 in Fig. 3, show that a logarithmic velocity distribution and the linear function (1) for $\varkappa = 0.40$–0.41 exist in fully developed turbulent pipe flow only in the shallow wall zone $y/R < 0.15$–0.2. At a greater distance from the wall the reduced mixing length l_0 deviates from the linear function (1). The maximum deviation (to 25%) is observed at $y/R = 0.5$–0.6. In the wall zone (at $y/R > 0.8$) the value of l_0 increases sharply.

It can be assumed that the indicated nonlinearity of the reduced mixing length is induced mainly by transfer of the kinetic energy of turbulent flucutations into the axial region of the pipe. We know [11] that such energy transfer is not covered by Prandtl's hypothesis, a fact that is also apparently manifested in the nature of the observed distribution of l_0.

According to a hypothesis of Kolmogorov [12], the turbulent viscosity ν_T is directly related to the turbulence kinetic energy E_T:

$$\nu_T = \sqrt{E_T/\rho}\, l_K. \tag{9}$$

A weaker dependence of the turbulence scale l_K on the transfer of turbulent fluctuation kinetic energy can be expected in this case.

A comparison of Eq. (9) with the turbulent viscosity equation according to Prandtl's hypothesis [1] enables us to deduce a relation between the Kolmogorov turbulence scale and the Prandtl mixing length:

$$l_{K0}/l_0 = \sqrt{\tau_T/E_T}\,. \tag{10}$$

Here we have introduced the concept of the reduced turbulence scale l_{K0}, which is defined according to an equation similar to (8): $l_{K0} = l_K\sqrt{\tau_w/\tau}$.

Substituting the values of τ_T/E_T calculated according to the experimental data of [13] in Eq. (10), we can determine the distribution of l_{K0}. The results of the calculations, which are represented by curves 4 and 5 in Fig. 3, show that the reduced turbulence scale l_{K0} varies essentially linearly over a wide range of values of y/R. The slopes of the straight lines in the middle interval ($0.2 < y/R < 0.8$) are somewhat smaller than in the interval of the logarithmic velocity distribution.

We have thus established the fact that the laws governing the variation of the turbulence scale l_K and the mixing length l in fully developed turbulent pipe flow are based on a linear function. The deviation of these quantities from the linear law (1) at a relative distance $y/R > 0.15$ is a consequence of the variation of the tangential stress and the transfer of turbulence kinetic energy along the radius of the pipe. The nonlinearity of the mixing length in the zone next to the wall (for $y_+ < 60$) is attributable to the process of direct dissipation of the average flow energy as heat.

NOTATION

A_+, dimensionless parameter in the Van Driest exponential equation; e, base of the natural logarithm; E_T, kinetic energy of turbulent fluctuations, J/m^3; l, mixing length, m; l_0, reduced mixing length, m; l_K, Kolmogorov turbulence scale, m; l_{K0}, reduced turbulence scale, m; R, inside radius of pipe, m; Re, Reynolds number; u, velocity of medium, m/sec; $u_* = \sqrt{\tau_w/\rho}$, dynamic velocity, m/sec; u', longitudinal velocity fluctuation, m/sec; v', transverse velocity fluctuation, m/sec; y, distance from the wall along the normal to it, m; $y_+ = yu_*/\nu$, dimensionless distance; $l_+ = lu_*/\nu$, dimensionless mixing length; $u_+ = u/u_*$, dimensionless velocity; $\varkappa$, dimensionless constant in the Prandtl linear equation; ρ, density of the medium, kg/m^3; ν, kinematic viscosity coefficient, m^2/sec; ν_T, turbulent viscosity coefficient (turbulent analog of kinematic viscosity coefficient), m^2/sec; τ, tangential stress, Pa; τ_w, wall tangential stress, Pa; τ_T, turbulent tangential stress, Pa.

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